Physics on the Celestial

Author: Bufan Zheng
Advisor: Yijian Du

Wuhan University

whuzbf@qq.com

January 2, 2024

Overview

Lorentz Group & Poincaré group

Definition

Lorentz group (O(1,3)) is the isometric transformations in Minkowski spacetime

Four disconnected parts:

- Proper orthochronous lorentz group: $SO(1,3)^{\uparrow}$
- Discret Transformation: $\mathcal{P}, \mathcal{T}$

We just need to care about $SO(1,3)^{\uparrow}$.

Definition

$$\text{Poincaré Group} = SO(1,3)^{\uparrow} \ltimes \mathbb{R}^{1,3}$$

It's the biggest symmetry group of 1+3 spacetime.

Double Cover

As we all know:

$$SO(3) \cong SU(2)/\mathbb{Z}_2 \tag{1}$$

Similarly:

$$SO(3,1)^{\uparrow} \cong SL(2,\mathbb{C})/\mathbb{Z}_2 \tag{2}$$

More fun with identities:

$$SO(2,1)^{\uparrow} \cong SL(2,\mathbb{R})/\mathbb{Z}_2, \quad SO(5,1)^{\uparrow} \cong SL(2,\mathbb{H})/\mathbb{Z}_2, \quad SO(9,1)^{\uparrow} \cong SL(2,\mathbb{O})/\mathbb{Z}_2 \tag{3}$$

Because

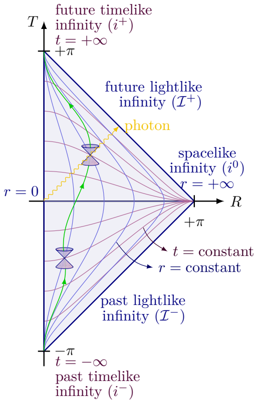
Theorem (Hurwitz)

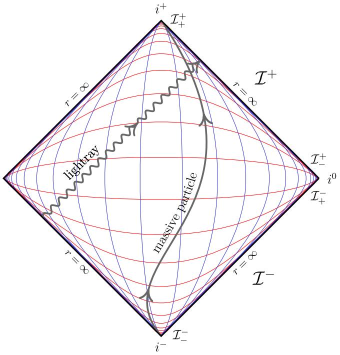
The normed division algebras over $\mathbb{R}$ are precisely $\mathbb{R}, \mathbb{C}, \mathbb{H}$ and $\mathbb{O}$.

So we can never find ternary number!

Penrose Diagrams ($Mink_4$)

Penrose diagram conformally conpactify the spacetime, represent it by a finite scale diagram.





Definition

Every point of $\mathcal{I}^{\pm}$ represent a sphere of infinite radius, called **Celestial Sphere** ($\mathcal{CS}^2$)

- Because $\mathcal{CS}^2$ can be seen as a complex plane after single-point compecting, we can use $(z, \bar{z})$ to label its angular coordinates ($\mathcal{S}^2 \cong \mathbb{C} \cup \{\infty\}$). We prefer to use retarded coordinates $(u, r, z, \bar{z})$ parametrize $\mathcal{I}^{+}$, and advanced coordinates $(v, r, z, \bar{z})$ on $\mathcal{I}^{-}$. Remember, The angle parameters $(z, \bar{z})$ are antipodally identity between $\mathcal{I}^{\pm}$.
- $SO(1,3)^{\uparrow} \cong SL(2,\mathbb{C})/\mathbb{Z}_2$, Lorentz transformations generate conformal transformations on the celestial sphere!

$$z \mapsto \frac{az+b}{cz+d}, \quad \bar{z} \mapsto \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}} \tag{4}$$

We can use infinite symmetries on CFT_2 to analyse scattering process in Mink_4

Conformal Basis

In QFT, we usually use fourier transformation to work in moment space, which can manifest the translational symmetry of Mink_d. To use the conformal symmerty of CFT_{d-2}, we can work with **Conformal Basis**:

- Photons:

$$A_{\mu a}^{\Delta,\pm}(X^\mu;\vec{w}) = -\frac{1}{(-q\cdot X \mp i\epsilon)^{\Delta-1}}\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial w^a}\log(-q\cdot X \mp i\epsilon) \tag{5}$$

- Gravitons:

$$h_{\mu_1\mu_2;a_1a_2}^{\Delta,\pm}(X;\vec{w}) = P_{a_1a_2}^{b_1b_2}\frac{1}{(-q\cdot X \mp i\epsilon)^{\Delta-2}}\partial_{b_1}\partial_{\mu_1}\log(-q\cdot X \mp i\epsilon)\partial_{b_2}\partial_{\mu_2}\log(-q\cdot X \mp i\epsilon) \tag{6}$$

where $P_{a_1a_2}^{b_1b_2} \equiv \delta_{(a_1}^{b_1}\delta_{a_2)}^{b_2} - \frac{1}{d}\delta_{a_1a_2}\delta^{b_1b_2}$, $\Delta \in \frac{d}{2} + i\mathbb{R}$.

However, in the soft region, which forced $\Delta = 1$, the conformal wave functions need to be redefined. (arXiv: 1810.05219)

For example, we can transform momentum space amplitudes to conformal space amplitudes:

$$\widetilde{\mathcal{A}}(\Delta_i, \vec{w}_i) \equiv \prod_{k=1}^{n} \int_{\mathrm{H}_{d+1}} [d\hat{p}_k] G_{\Delta_k}(\hat{p}_k; \vec{w}_k) \mathcal{A}(\pm m_i \hat{p}_i^{\mu}) \tag{7}$$

For massless particles scattering, this transformation is **Mellin transoformation**.
3 massive scalar scattering (ϕ^3-theory) at tree-level ($m_{\mathrm{in}} = (2+\epsilon) m_{\mathrm{out}}$):

$$\widetilde{\mathcal{A}} = \frac{i 2^{\frac{9}{2}} \pi^6 \lambda \Gamma(\frac{\Delta_1+\Delta_2+\Delta_3-2}{2}) \Gamma(\frac{\Delta_1+\Delta_2-\Delta_3}{2}) \Gamma(\frac{\Delta_1-\Delta_2+\Delta_3}{2}) \Gamma(\frac{\Delta_1-\Delta_2+\Delta_3}{2}) \Gamma(\frac{-\Delta_1+\Delta_2+\Delta_3}{2}) \sqrt{\varepsilon}}{m^4 \Gamma(\Delta_1) \Gamma(\Delta_2) \Gamma(s_3) |w_1 - w_2|^{\Delta_1+\Delta_2-\Delta_3} |w_2 - w_3|^{\Delta_2+\Delta_3-\Delta_1} |w_3 - w_1|^{\Delta_3+\Delta_1-\Delta_2}} + \mathcal{O} \tag{8}$$

But we need a more systematic approach, CCFT is comming!

Infrared Triangle

Surprisingly, three seemingly unrelated areas can be linked by some kind of relationship!

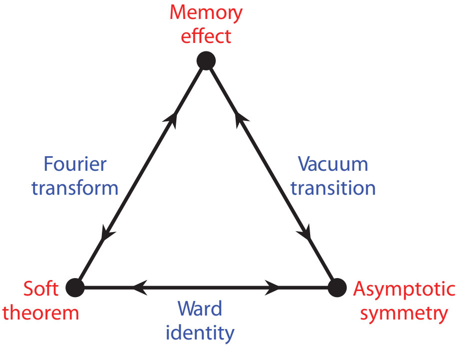


Figure: The infrared triangle

Large gauge symmetry $\not\subseteq$ Gauge symmetry

Definition

$$\text{Asymptotic symmetries} = \frac{\text{Allowed symmetries}}{\text{Trivial symmetries}}$$

We can use Asymptotic analyse on QED and perturbative gravity to find asymptotic symmetries, called Large gauge/diffeomorphic symmetry. Noether theorem tells us there must be some conserved charges correspond to asymptotic symmetries, in fact, there are infinite number of conserved charges! These symmetries will spontaneous breaking and bring soft photons and soft gravitons to our world.

$$\langle \mathrm{out}| \left(Q_\varepsilon^+ \mathcal{S} - \mathcal{S} Q_\varepsilon^- \right) |\mathrm{in}\rangle = 0 \iff \sum_{k=1}^{m} \frac{eQ_k^{\mathrm{out}} p_k^{\mathrm{out}} \cdot \varepsilon}{p_k^{\mathrm{out}} \cdot q} - \sum_{k=1}^{n} \frac{eQ_k^{\mathrm{in}} p_k^{\mathrm{in}} \cdot \varepsilon}{p_k^{\mathrm{in}} \cdot q} \tag{9}$$

Asymptotic symmetries $\overset{\text{Ward Indentity}}{\iff}$ Soft theorem!

Open Questions

There are some open questions:

- CCFT
- Non-Abelian promotion
- Supersymmetry on the celestial
- BCFW, CHY, KLT relations of the celestial $\mathcal{A}$mplitudes
- ...

There's a lot of new physics on the celestial waiting to be discovered !

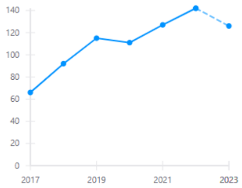
Lectures on the Infrared Structure of Gravity and Gauge Theory

Andrew Strominger (Harvard U.)
Mar 15, 2017

158 pages
ISBN: 9780691179735
e-Print: 1703.05448 [hep-th]
View in: ADS Abstract Service

pdf links cite claim reference search 779 citations

Citations per year



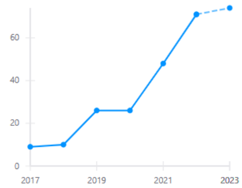
Conformal basis for flat space amplitudes

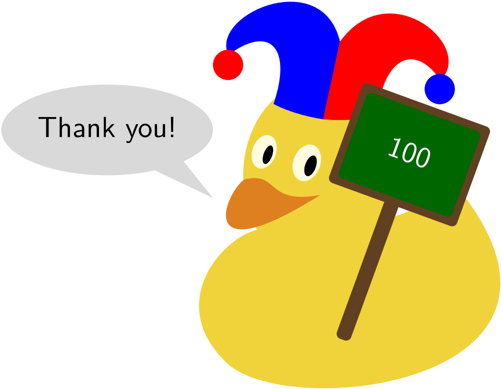
Sabrina Pasterski (Harvard U.), Shu-Heng Shao (Princeton, Inst. Advanced Study)
May 2, 2017

17 pages
Published in: *Phys.Rev.D* 96 (2017) 6, 065022
Published: Sep 25, 2017
e-Print: 1705.01027 [hep-th]
DOI: 10.1103/PhysRevD.96.065022
View in: AMS MathSciNet, ADS Abstract Service

pdf cite claim reference search 264 citations

Citations per year




Thank you!
100