The CHY Formalism for Massless Scattering

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Overview

Feynman Diagrams

- For **QED** process, Feynman diagram is a efficient tool to calculate scattering amplitudes.

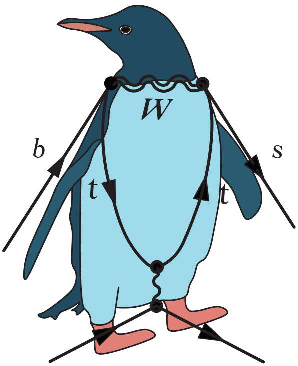
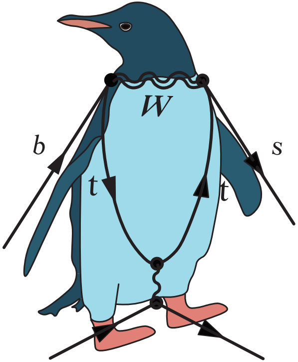


Figure: Kawaii feynman diagram

Feynman Diagrams

- For **QED** process, Feynman diagram is a efficient tool to calculate scattering amplitudes.
- Feynman's rule is derived from the Lagrangian, there are many terms in the Lagrangian that are blame to **gauge redundancy**, which inevitably leads to very complex expressions for individual Feynman diagrams, but the superposition of all Feynman diagrams is simple.

$$S_{\text{EH}} = \int d^D x \left[h\partial^2 h + \kappa h^2 \partial^2 h + \kappa^2 h^3 \partial^2 h + \kappa^3 h^4 \partial^2 h + \cdots \right]$$



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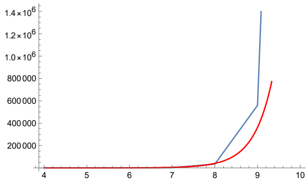

Figure: Too many diagrams to sum

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- We have too many Feynman diagrams to sum. The number of diagrams is growing much **faster** than $n!$.

Chinese Magic

Spinor-Helicity Formalism

- Using On-shell condition $p^2 = 0$:

$$p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}} p_{\mu} = \lambda_a \tilde{\lambda}_{\dot{a}} \equiv |p]\,\langle p|$$

  we can do the same thing for $\bar{\sigma}^{\mu}_{\dot{a}a} p_{\mu}$ to define $|p\rangle$ and $[p|$.

- It is easy to see that $|\cdot\rangle$ and $|\cdot]$ automatically satisfy the gauge invariant condition:

$$p^{\dot{a}b}|p]_b = 0, \quad p_{a\dot{b}}|p\rangle^{\dot{b}} = 0, \quad [p|^b p_{b\dot{a}} = 0, \quad \langle p|_{\dot{b}} p^{\dot{b}a} = 0.$$

- In fact we can use gauge degree of freedom to simplify our expressions:

$$\epsilon^{+}_{\mu}(k;q) = \frac{\langle q^{-}|\gamma_{\mu}|k^{-}]}{\sqrt{2}\langle qk\rangle}$$

  The reference momenta q can be chosen freely.

Tree Amplitudes of YM

- MHV Amplitudes (Park-Taylor):

$$A_n\left[1^+\ldots i^-\ldots j^-\ldots n^+\right]=\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- N^kMHV Amplitudes:

$$A_n^{\mathrm{NPMHV}}(c_0,c_1,\ldots,c_p,n)=\frac{\delta^{(4)}(p)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}\times$$

$$\times\sum_{\text{all paths of length }p}1\cdot\tilde{R}_{n;a_1b_1}\cdot\tilde{R}^{\{L_2\};\{U_2\}}_{n;\{I_2\};a_2b_2}\cdot\ldots\cdot\tilde{R}^{\{L_p\};\{U_p\}}_{n;\{I_p\};a_pb_p}$$

$$\times\left(\det\Xi_n^{\mathrm{path}}(c_0,\ldots,c_p)\right)^4$$

Complex, but fully solvable by computers

Rimann Sphere

Momentum Space

$$\mathfrak{K}_{D,n} := \{(k_1^\mu, k_2^\mu, \ldots, k_n^\mu) | \sum_{a=1}^{n} k_a^\mu = 0, k_1^2 = k_2^2 = \cdots = k_n^2 = 0\}/SO(1, D-1) \quad (1)$$

If there is no codimensional singularity

$$s_{a_1,a_2,\ldots,a_r} := (k_{a_1} + k_{a_2} + \cdots + k_{a_r})^2 \neq 0, \quad \forall r = 1, \ldots, n \quad (2)$$

We can consider the moduli space of Riemann spheres $\mathbb{CP}^1$ with n distinct punctures on it to carve $\mathfrak{K}_{D,n}$ equivalently.

$$\mathfrak{M}_{0,n} \equiv \{\sigma_1, \sigma_2, \ldots, \sigma_n\}/SL(2, \mathbb{C})$$

$$\mathfrak{K}_{D,n} \iff \mathfrak{K}_{D,n} \text{ by } k_a^\mu = \frac{1}{2\pi i} \oint_{|z-\sigma_a|=\epsilon} dz \frac{p^\mu(z)}{\prod_{b=1}^{n}(z-\sigma_b)} \quad (3)$$

Scattering Equations

Using eq.3, we can derive the scattering equation:

$$\boxed{\sum_{b\neq a}\frac{s_{ab}}{\sigma_a-\sigma_b},\qquad a=1,2,\ldots,n,\qquad s_{ab}=2k_a\cdot k_b} \tag{4}$$

- n equations but only $n-3$ of them are independent.

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- KLT orthogonality of solutions to scattering equations.

$$\frac{(i,j)}{(i,i)^{\frac{1}{2}}(j,j)^{\frac{1}{2}}}=\delta_{ij} \tag{5}$$

Where,

$$(i,j):=\sum_{\alpha,\beta\in S_{n-3}} V^{(i)}(\alpha)S[\alpha|\beta]U^{(j)}(\beta)$$

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  Where,

$$V(\omega)=\frac{1}{(\sigma_1-\sigma_{\omega(2)})(\sigma_{\omega(2)}-\sigma_{\omega(3)})\cdots(\sigma_{\omega(n-2)}-\sigma_{n-1})(\sigma_{n-1}-\sigma_n)(\sigma_n-\sigma_1)},$$

$$U(\omega)=\frac{1}{(\sigma_1-\sigma_{\omega(2)})(\sigma_{\omega(2)}-\sigma_{\omega(3)})\cdots(\sigma_{\omega(n-2)}-\sigma_{n})(\sigma_{n}-\sigma_{n-1})(\sigma_{n-1}-\sigma_1)}.$$

Bi-adjoint Scalar, YM and Einstein Gravity

The amplitudes of many QFTs (at the tree level) can be expressed by a unified formula (Cachazo, He, Yuan, 2013).

$$\boxed{\mathcal{A}_n = \int d\mu_n \mathcal{I}_n, \quad d\mu_n = \frac{d^n\sigma}{\mathrm{volSL}(2,\mathbb{C})} \prod_a {}' \delta\left(\sum_{b\neq a} \frac{s_{ab}}{\sigma_{ab}} \right)} \tag{6}$$

For different theories, the integral measure is the same, differing only in the CHY integrands $\mathcal{I}_n$

$$\mathcal{M}_n^{(\mathbf{s})} = \int \frac{d^n\sigma}{\mathrm{vol\,SL}(2,\mathbb{C})} \prod_a {}' \delta(\sum_{b\neq a} \frac{s_{ab}}{\sigma_a - \sigma_b}) \left(\frac{\mathrm{Tr}(T^{\mathbf{a}_1} T^{\mathbf{a}_2} \cdots T^{\mathbf{a}_n})}{(\sigma_1 - \sigma_2)\cdots(\sigma_n - \sigma_1)} + \ldots \right)^{2-\mathbf{s}} (\mathrm{Pf}'\,\Psi)^{\mathbf{s}} \tag{7}$$

For Scalar, $s = 0$,

$$\mathcal{L}^{\Phi^3} := -\frac{1}{2}\partial_\mu \Phi_{I,\tilde{I}} \partial^\mu \Phi^{I,\tilde{I}} - \frac{\lambda}{3!} f_{I,J,K} \tilde{f}_{\tilde{I},\tilde{J},\tilde{K}} \Phi^{I,\tilde{I}} \Phi^{J,\tilde{J}} \Phi^{K,\tilde{K}} \tag{8}$$

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For Scalar, $s = 0$,

$$\mathcal{I}^{\Phi^3}_{U(N)\times U(\tilde{N})} := \mathcal{C}_{U(N)}\mathcal{C}_{U(\tilde{N})} \tag{8}$$

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Where

$$\mathcal{C}_{\mathrm{U}(N)} := \sum_{\alpha \in S_n/\mathbb{Z}_n} \mathrm{tr}(T^{\alpha(1)} T^{\alpha(2)} \ldots T^{\alpha(n)})\, \text{PT}_n(\alpha)$$

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Where,

$$\mathrm{PT}_n[\alpha] := \frac{1}{\sigma_{\alpha(1)\alpha(2)}\sigma_{\alpha(2),\alpha(3)}\cdots\sigma_{\alpha(n),\alpha(1)}}$$

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Delta functions in eq.6 totally local the integral, so we don't need to calculate annoying integral, we just need to solve the scattering equations.

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i.e.

$$\sum_{\{\sigma\}\in\text{solutions}} \frac{(\sigma_{pq}\sigma_{qr}\sigma_{rp})(\sigma_{ij}\sigma_{jk}\sigma_{ki})}{|\Phi|^{ijk}_{pqr}} \mathcal{I} \tag{8}$$

Bi-adjoint Scalar, YM and Einstein Gravity

For YM, $s = 1$, (*Gervais–Neveu gauge, ghost free*)

$$\mathcal{L} = \mathrm{Tr}\left(-\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - i\sqrt{2}g\partial^\mu A^\nu A_\nu A_\mu + \frac{g^2}{4}A^\mu A^\nu A_\nu A_\mu\right)$$

$$\mathcal{I}_n^{\mathsf{YM}} = \mathcal{C}_n \mathrm{Pf}'\Psi(\{k, \epsilon, \sigma\}) \tag{9}$$

For Gravitons, $s = 2$, (*de-Donder gauge, ghost free*)

$$\begin{aligned}\mathcal{L}_{\mathrm{EH}} =& \partial_\alpha h \partial_\beta h^{\alpha\beta} - \partial_\alpha h_{\beta\gamma}\partial^\beta h^{\alpha\gamma} - \frac{1}{2}(\partial_\alpha h)^2 + \frac{1}{2}(\partial_\gamma h_{\alpha\beta})^2 + \mathcal{O}\left(\kappa, h^3\right) \\ &+ \partial^\nu h_{\mu\nu}\partial^\rho h^\mu_\rho + \frac{1}{4}(\partial_\mu h)^2 - \partial^\nu h_{\mu\nu}\partial^\mu h\end{aligned}$$

$$\mathcal{I}_n^{\mathsf{GR}} = \mathrm{Pf}'\Psi_n \mathrm{Pf}'\tilde{\Psi}_n \tag{10}$$

We introduce $\tilde{\Psi}$, because generally, we can contain dilatons and B-fields in GR. For pure graviton scattering, $\Psi = \tilde{\Psi}$.

Bi-adjoint Scalar, YM and Einstein Gravity

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For Gravitons, $s = 2$,

$$\mathcal{I}_n^{\text{GR}} = \text{Pf}' \Psi_n \text{Pf}' \tilde{\Psi}_n \tag{10}$$

Where,

$$\Psi = \begin{pmatrix} A & -C^T \\ C & B \end{pmatrix}, \quad \text{Pf}' \Psi := \frac{(-1)^{i+j}}{(\sigma_i - \sigma_j)} \text{Pf}(\Psi_{ij}^{ij}) \tag{11}$$

and,

$$A_{ab} = \begin{cases} \frac{s_{ab}}{\sigma_a - \sigma_b}, & a \neq b \\ 0, & a = b, \end{cases} \quad B_{ab} = \begin{cases} \frac{2\varepsilon_a \cdot \varepsilon_b}{\sigma_a - \sigma_b}, & a \neq b \\ 0, & a = b, \end{cases} \quad C_{ab} = \begin{cases} \frac{2\epsilon_a \cdot k_b}{\sigma_a - \sigma_b}, & a \neq b \\ -\sum_{c \neq a} \frac{2\epsilon_a \cdot k_c}{\sigma_a - \sigma_c}, & a = b \end{cases} \tag{12}$$

More theories and their connections

- The greatest advance in scattering amplitudes in the last two decades has been the formulation of the BCFW recursion relations, which has greatly simplified the calculations.

$$A_n = \sum_{\text{diagrams } I} \hat{A}_{\mathrm{L}}(z_I)\,\frac{1}{P_I^2}\,\hat{A}_{\mathrm{R}}(z_I)$$

$$= \sum_{\text{diagrams } I}$$

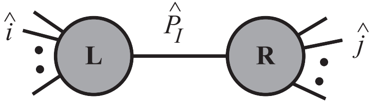

Figure: BCFW Recursion Relations.

More theories and their connections

- Scattering equations are very difficult to find all (n-3)! solutions. In this sense, the CHY formula doesn't bring us a new efficient tool for calculating amplitudes. But it gives us a unified framework to consider connections between different theories.

Theory	Integrand	Section
Einstein gravity	$\mathrm{Pf}'\Psi_n \, \mathrm{Pf}'\Psi_n$	4.5
Yang–Mills	$\mathcal{C}_n \, \mathrm{Pf}'\Psi_n$	4.4.1
Φ^3 flavored in $U(N) \times U(\tilde{N})$	$\mathcal{C}_n \, \mathcal{C}_n$	4.2.1
Einstein–Maxwell	$\mathrm{Pf}[\mathcal{X}_n]_\gamma \, \mathrm{Pf}'[\Psi_n]_{:\hat{\gamma}} \, \mathrm{Pf}'\Psi_n$	5.1.3
Einstein–Yang–Mills	$\mathcal{C}_{\mathrm{tr}_1} \cdots \mathcal{C}_{\mathrm{tr}_t} \, \mathrm{Pf}'\Pi(\mathrm{h}; \mathrm{tr}_1 \ldots, \mathrm{tr}_t) \, \mathrm{Pf}'\Psi_n$	5.2
Yang–Mills–Scalar	$\mathcal{C}_n \, \mathrm{Pf}[\mathcal{X}_n]_\mathrm{s} \, \mathrm{Pf}'[\Psi_n]_{:\hat{\mathrm{s}}}$	5.1.1
generalized Yang–Mills–Scalar	$\mathcal{C}_n \, \mathcal{C}_{\mathrm{tr}_1} \cdots \mathcal{C}_{\mathrm{tr}_t} \, \mathrm{Pf}'\Pi(\mathrm{g}; \mathrm{tr}_1 \ldots, \mathrm{tr}_t)$	5.2.4
Born–Infeld	$\mathrm{Pf}'\Psi_n \, (\mathrm{Pf}' A_n)^2$	4.4.3
Dirac–Born–Infeld	$\mathrm{Pf}[\mathcal{X}_n]_\mathrm{s} \, \mathrm{Pf}'[\Psi_n]_{:\hat{\mathrm{s}}} \, (\mathrm{Pf}' A_n)^2$	5.1.2
extended Dirac–Born–Infeld	$\mathcal{C}_{\mathrm{tr}_1} \cdots \mathcal{C}_{\mathrm{tr}_t} \, \mathrm{Pf}'\Pi(\gamma; \mathrm{tr}_1 \ldots, \mathrm{tr}_t) \, (\mathrm{Pf}' A_n)^2$	5.2.5
$U(N)$ non-linear sigma model	$\mathcal{C}_n \, (\mathrm{Pf}' A_n)^2$	4.2.3
special Galileon	$(\mathrm{Pf}' A_n)^4$	4.2.6

Figure: CHY integrands of different QFTs

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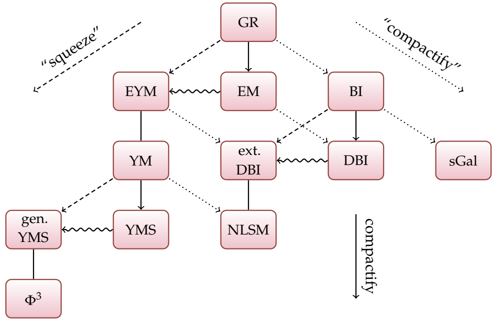


Connections among integrands. Compactify: ⟶. Squeeze: - - ->. "Compactify": ·······>. Non-Abelian: ⇝. Restrict to single trace: ——.

Figure: Theory Web.

KLT double copy

Gravity $=$ YM$^2/\phi^3$,

$$\boxed{M_n = (-1)^n \sum_{\beta,\gamma} \frac{A_n(1,\beta_{2,n-1},n)\widetilde{\mathcal{S}}[\beta_{2,n-1}|\gamma_{2,n-1}]_{p_n}\widetilde{A}_n(n,\gamma_{2,n-1},1)}{s_{23\ldots n}}} \tag{13}$$

Where, KLT kernel $\mathcal{S}[\alpha|\beta]$ can be constructed by double partial amplitudes of ϕ^3 theory:

$$S[\alpha|\beta] = \prod_{i=2}^{n-2} \left(s_{1,\alpha(i)} + \sum_{j=2}^{i-1} \theta(\alpha(j),\alpha(i))_\beta s_{\alpha(j),\alpha(i)} \right) = m(\alpha|\beta)^{-1} \tag{14}$$

So, we divide YM^2 by ϕ^3.

KLT relations

KLT othogonality eq.5 can be rewrited as,

$$\delta_{\alpha,\gamma} = \sum_{\beta \in S_{n-3}} \int d\mu_n \mathbf{PT}(\alpha)\mathbf{PT}(\beta) S[\beta|\gamma] \tag{15}$$

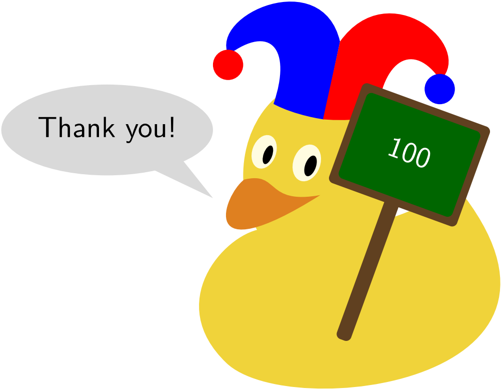
So if we have a theory $\mathcal{M}_n$ whose CHY integrand is $\mathcal{I} = \mathcal{I}^L \mathcal{I}^R$. Then we can define two partial amplitudes $\mathcal{M}_n^L$ and $\mathcal{M}_n^R$, whose CHY integrands are $\mathcal{I}^L \cdot \mathbf{PT}$ and $\mathcal{I}^R \cdot \mathbf{PT}$, respectively.

Which gives a general KLT relation,[1]

$$\mathcal{M}_n = \mathcal{M}_n^L \otimes_{\mathrm{KLT}} \mathcal{M}_n^R \tag{16}$$

The CHY formalism enables us to gain insights that would otherwise be difficult to discern from the Lagrangian.

[1] "KLT" suffix means we need a KLT kernel.


Thank you!
100